Week 11: The vented cabinet loudspeaker (part 2)

Microphone and Loudspeaker Design - Level 5

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What are we covering today?

1. Vented cabinet equivalent circuit

2. Choosing an alignment

3. Passive radiator loudspeaker

Vented cabinet equivalent circuit

Vented cabinet loudspeaker: box volume velocity

- Driver/diaphragm:

  M_{AS}, C_{AS}, R_{AS} - mechanical mass, compliance and damping (inc. air loading)

  U_D - volume velocity

- Cabinet:

  C_{AB}, R_{AB} - compliance and damping

  U_B - volume velocity

- Vent:

  M_{AV}, R_{AV} - mass and damping (inc. air loading)

  U_V - volume velocity

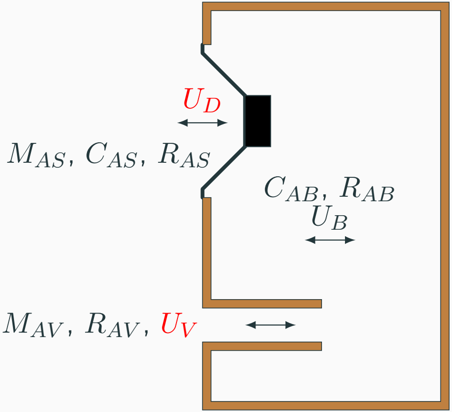


Figure 1: Vented cabinet loudspeaker.

Vented cabinet loudspeaker: equivalent circuit

- The supplied current U_D is separated into U_V and $U_B = U_D - U_V$
- To make life easier we can group all acoustic loading terms within the driver and vent mass/damping
- To design a vented loudspeaker we will need to analyse this circuit...

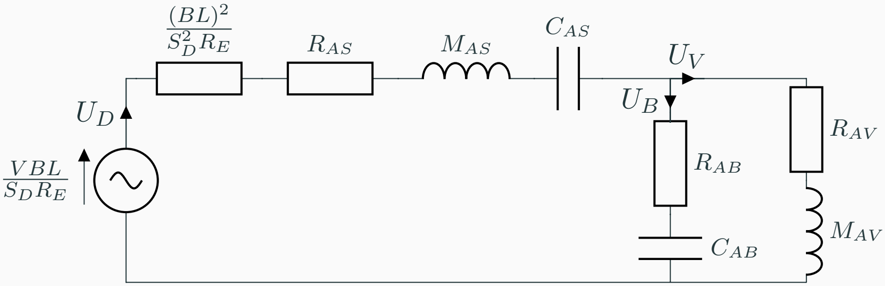


Figure 2: Simplified complete equivalent circuit for vented cabinet.

Vented cabinet loudspeaker: equivalent circuit analysis

- We have an equivalent circuit for the low frequency response of a vented cabinet.
- We can analyse it and figure out what sort of frequency response we will get for some specified parameters.
- **Big question:** how do we design a vented loudspeaker to achieve a *particular* frequency response shape?
  - Turns out its quite easy!
  - But first we need to play with some equations...

Vented cabinet loudspeaker: simplified circuit

$$Z_D = \frac{BL^2}{S_D^2 R_E} + R_{AD} + j\omega M_{AS} + \frac{1}{j\omega C_{AS}} \quad (1)$$

- Group into complex impedances
- Use AC circuit analysis

$$Z_B = R_{AB} + \frac{1}{j\omega C_{AB}} \qquad Z_V = R_{AV} + j\omega M_{AV} \quad (2)$$

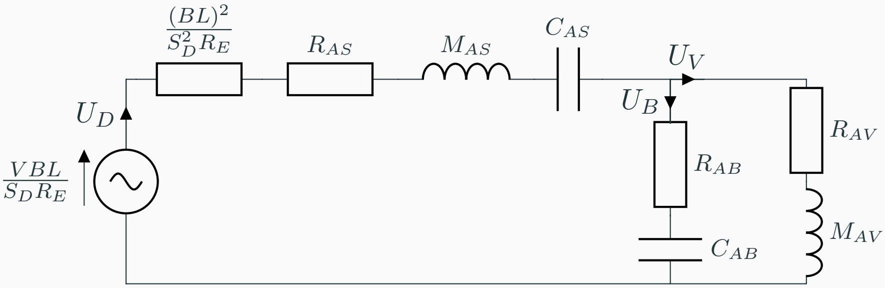


Figure 2: Simplified complete equivalent circuit for vented cabinet.

Vented cabinet loudspeaker: simplified circuit

- Total impedance of circuit:

$$Z_T = Z_D + \frac{Z_B Z_V}{Z_B + Z_V} \qquad (3)$$

- Get current through each branch:

$$U_D = \frac{\frac{VBL}{S_D R_E}}{Z_T} \qquad (4)$$

$$U_B = \frac{Z_V}{Z_B + Z_V} U_D \qquad (5)$$

$$U_V = \frac{Z_B}{Z_B + Z_V} U_D \qquad (6)$$

- In far field driver/vent path length difference is negligible. Total volume velocity:

$$U_T = U_D + (-)U_V = U_B \qquad (7)$$

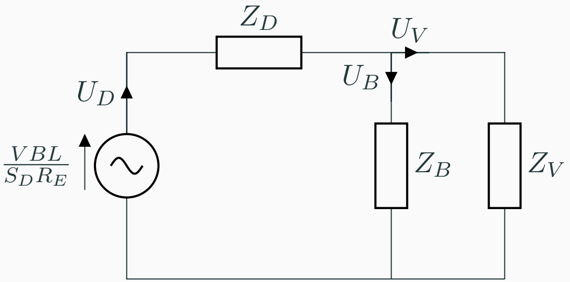


Figure 3: Grouped equivalent circuit.

Vented cabinet loudspeaker: simplified circuit

- The total volume velocity is the current (or volume velocity) through the box branch.

$$U_B = U_T = \frac{Z_V}{Z_B + Z_V} U_D \qquad (8)$$

$$U_T = \frac{Z_V}{Z_B + Z_V} \frac{\frac{VBL}{S_D R_E}}{Z_D + \frac{Z_B Z_V}{Z_B + Z_V}} \qquad (9)$$

- We want to simplify this volume velocity expression and substitute into (monopole) radiation model...

- After some algebra...

$$U_T = \frac{\left(\frac{VBL}{S_D R_E}\right)}{Z_D \left(\frac{Z_B}{Z_V} + 1\right) + Z_B} \qquad (10)$$

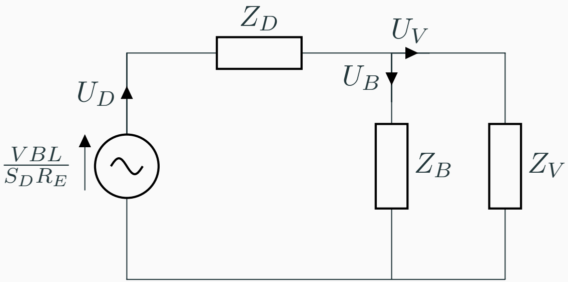


Figure 3: Grouped equivalent circuit.

Vented cabinet loudspeaker: simplified circuit

- Total volume velocity

$$U_T = \frac{\frac{VBL}{S_D R_E}}{Z_D\left(\frac{Z_B}{Z_V}+1\right)+Z_B} \tag{11}$$

- Substitute branch impedance terms (for now consider *lossless* cabinet)

$$Z_D = \overbrace{\frac{BL^2}{S_D^2 R_E} + R_{AD}}^{R_{AT}=R_{AS}} + j\omega M_{AS} + \frac{1}{j\omega C_{AS}} \tag{12}$$

$$Z_B = \cancel{R_{AB}} + \frac{1}{j\omega C_{AB}} \qquad Z_V = \cancel{R_{AV}} + j\omega M_{AV} \tag{13}$$

$$U_T = \frac{\frac{VBL}{S_D R_E}}{\left(R_{AS} + j\omega M_{AS} + \frac{1}{j\omega C_{AS}}\right)\left(\frac{1}{(j\omega)^2 M_{AV} C_{AB}}+1\right)+\frac{1}{j\omega C_{AB}}} \tag{14}$$

Vented cabinet loudspeaker: simplified circuit

- Factor out $j\omega M_{AS}$ (just like we did for sealed cabinet)

$$U_T = \frac{VBL}{j\omega M_{AS} S_D R_E} \frac{1}{\left(1 + \frac{R_{AS}}{j\omega M_{AS}} + \frac{1}{(j\omega)^2 M_{AS} C_{AS}}\right)\left(\frac{1}{(j\omega)^2 M_{AV} C_{AB}} + 1\right) + \frac{1}{(j\omega)^2 M_{AS} C_{AB}}} \tag{15}$$

- Re-parametrize in terms of free driver and cabinet properties

$$\frac{Q_{TS}}{\omega_s} = \frac{M_{AS}}{R_{AS}} \qquad \omega_s^2 = \frac{1}{M_{AS} C_{AS}} \qquad \omega_b^2 = \frac{1}{M_{AV} C_{AB}} \qquad \alpha = \frac{C_{AS}}{C_{AB}} \tag{16}$$

- Re-parametrize in terms of free driver and cabinet properties

$$U_T = \frac{VBL}{j\omega M_{AS} S_D R_E} \frac{1}{\left(1 + \frac{\omega_s}{j\omega}\frac{1}{Q_{TS}} + \left(\frac{\omega_s}{j\omega}\right)^2\right)\left(\left(\frac{\omega_b}{j\omega}\right)^2 + 1\right) + \alpha \left(\frac{\omega_s}{j\omega}\right)^2} \tag{17}$$

Vented cabinet loudspeaker: simplified circuit

- Total volume velocity product of two terms

$$U_T = \overbrace{\frac{VBL}{j\omega M_{AS} S_D R_E}}^{\text{First order LP}} \overbrace{\frac{1}{\left(1 + \frac{\omega_s}{j\omega}\frac{1}{Q_{TS}} + \left(\frac{\omega_s}{j\omega}\right)^2\right)\left(\left(\frac{\omega_b}{j\omega}\right)^2 + 1\right) + \alpha\left(\frac{\omega_s}{j\omega}\right)^2}}^{\text{Fourth order HP: } F(j\omega)} \tag{18}$$

- This looks very similar to sealed cabinet, except $E(j\omega) \rightarrow F(j\omega)$
- Substitute into monopole radiation equation

$$p(r) = \frac{j\omega\rho_0}{4\pi r} U_T = \frac{\cancel{j\omega}\rho_0 VBL}{\cancel{j\omega} 4\pi r M_{AS} S_D R_E} F(j\omega) \tag{19}$$

- **All frequency dependence is contained within the $F(j\omega)$ term** - remaining terms control sensitivity

Choosing an alignment

Vented cabinet loudspeaker: simplified circuit

- Frequency response of vented cabinet is governed by the equation:

$$F(j\omega) = \frac{1}{\left(1 + \frac{\omega_s}{j\omega}\frac{1}{Q_{TS}} + \left(\frac{\omega_s}{j\omega}\right)^2\right)\left(\left(\frac{\omega_b}{j\omega}\right)^2 + 1\right) + \alpha\left(\frac{\omega_s}{j\omega}\right)^2} \tag{20}$$

- Now, given a specific driver (ω_s, Q_{TS}), how do we design a vented cabinet?
  1. Play with α and ω_b parameters until target response is obtained...
  2. Do lots of maths and, using filter design theory, derive exact values...
  3. Thiele and Small to the rescue..!
- Note, the design of a vented cabinet is more complex than a sealed cabinet where we just chose the required Q_{TC} and solved for the cabinet volume...

Thiele and Small: Australian loudspeaker wizards

- For a long time it was known that ports can improve low frequency performance - design was generally trial and error
- Thiele recognised that the driver loudspeaker acts like a high pass filter - possible to apply filter design methods directly to the design of loudspeakers
- Small (a student of Thiele) published a series of papers which provided a ‘fool proof’ way of designing vented cabinet loudspeakers - still used today!
- Important aspect of their work was to realise the driver itself was an important design parameter in the design process.

Figure 4: Neville Thiele and Richard Small.

Vented cabinet loudspeaker: simplified circuit

- Thiele noticed that the vented loudspeaker frequency response

$$F(j\omega) = \frac{1}{\left(1 + \frac{\omega_s}{j\omega}\frac{1}{Q_{TS}} + \left(\frac{\omega_s}{j\omega}\right)^2\right)\left(\left(\frac{\omega_b}{j\omega}\right)^2 + 1\right) + \alpha\left(\frac{\omega_s}{j\omega}\right)^2} \tag{21}$$

  has the same form as a general fourth order electrical network/filter

$$G_{HP}(j\omega) = \frac{1}{1 + a_1\left(\frac{1}{j\omega}\right)\frac{1}{T_0} + a_2\left(\frac{1}{j\omega}\right)^2\frac{1}{T_0^2} + a_3\left(\frac{1}{j\omega}\right)^3\frac{1}{T_0^3} + \left(\frac{1}{j\omega}\right)^4\frac{1}{T_0^4}} \tag{22}$$

- Constants a_1, a_2, and a_3 define the shape of the response curve and $T_0 = (\omega_s\omega_b)^{1/2}$ is filter time constant. Can get these from electrical network theory!

Thiele and Small: seminal papers

- If you are interested in the design of loudspeakers you should read these seminal papers by Thiele and Small.
- A. N. Thiele, *Loudspeakers in Vented Boxes*, JAES, Vol. 19, Num. 5/6, 1971
  - Part 1 (synthesis approach and system alignments)
  - Part 2 (conclusions pertinent to efficiency, driver Q, and box volume)
- R. H. Small, *Vented-box Loudspeaker Systems*, JAES, Vol. 21, Num. 5-8, 1973
  - Part 1: Small-signal analysis
  - Part 2: Large signal analysis
  - Part 3: Synthesis
  - Part 4: Appendices
- We will be using the design charts presented in Part 1 of Small's paper.

Small charts: foolproof vented cabinet design

- α is the compliance ratio $\alpha = C_{AS}/C_{AB}$
- Q_{TS} is the total compliance of the driver
- h is the ratio of free driver and Helmholtz resonance $h = f_b/f_s$
- f_3/f_s is the ratio of free driver resonance and -3 dB cut-off freq.
- B and k are parameters that describe the shape of frequency response
- Chart is obtained by specifying cabinet damping in terms of Q factor.

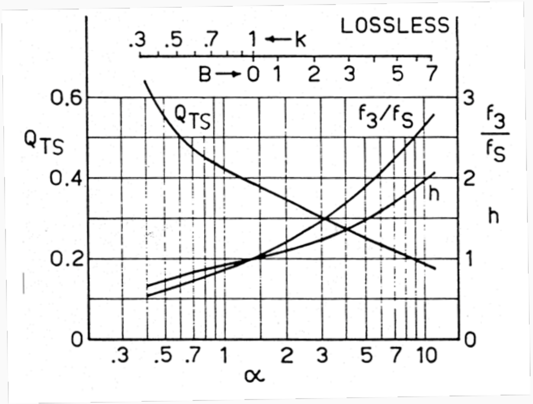


Figure 5: Design chart for **lossless** cabinet.

Small charts: foolproof vented cabinet design

1. Select driver $\rightarrow Q_{TS}, f_s, C_{AS}$
2. Look up Q_{TS} on left axis, find α, calculate volume $V_{AB} = C_{AB}\rho_0 c^2$
3. For the same value of α, find h, calculate Helmholtz frequency
4. Use f_b determine the vent mass M_{AV} (then pick port dimensions)
5. Find -3 dB cut-off frequency using f_3/f_s

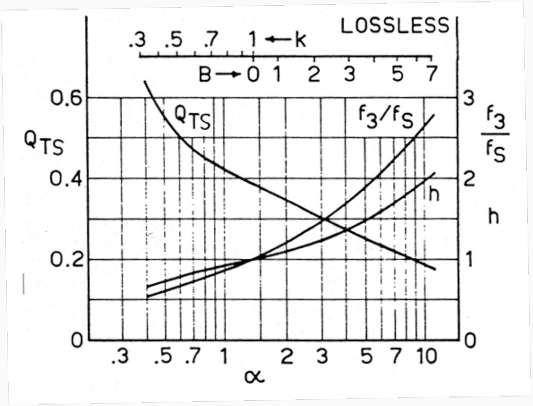


Figure 5: Design chart for **lossless** cabinet.

Small charts: different cabinet damping

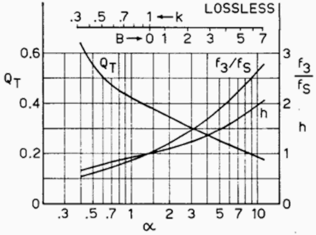


Fig. 6. Alignment chart for lossless vented-box systems.

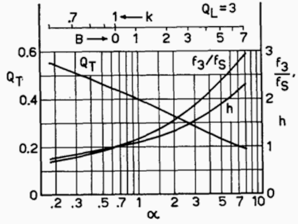


Fig. 13. Alignment chart for vented-box systems with $Q_B = Q_L = 3$.

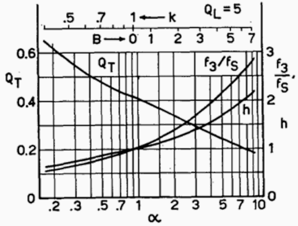


Fig. 12. Alignment chart for vented-box systems with $Q_B = Q_L = 5$.

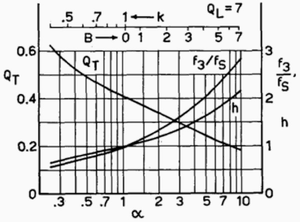


Fig. 11. Alignment chart for vented-box systems with $Q_B = Q_L = 7$.

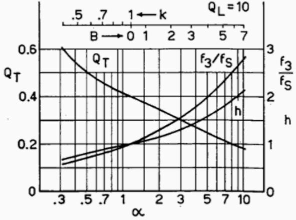


Fig. 10. Alignment chart for vented-box systems with $Q_B = Q_L = 10$.

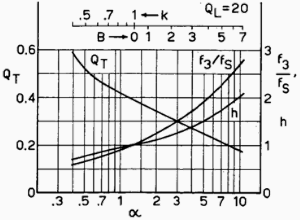


Fig. 9. Alignment chart for vented-box systems with $Q_B = Q_L = 20$.

Small charts: effect of misalignment

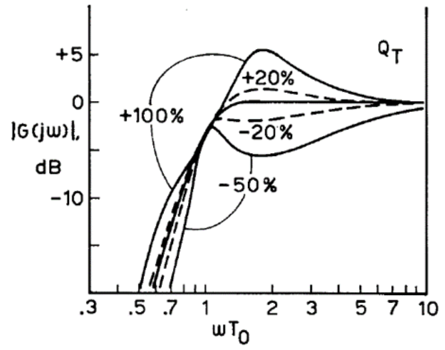
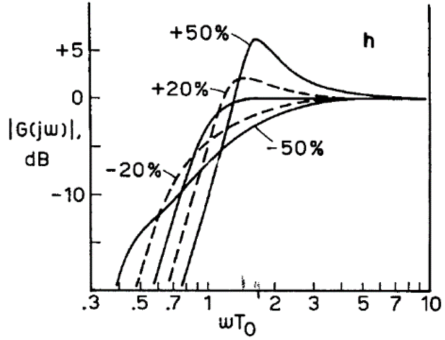


Fig. 7. Variations in frequency response of lossless B4-aligned vented-box system for misalignment of Q_T (from simulator).



· Fig. 8. Variations in frequency response of lossless B4-aligned vented-box system for misalignment of h (from simulator).

Figure 6: Effect of misalignment on system performance (*Small, Part 1*).

Sealed vs. Vented

- **Sealed:**
  - Easier to design / more forgiving to inaccurate parameters
  - Better transient response
  - Lower efficiency than ported
  - Less extended bass response
  - High velocity at driver resonance
- **Vented:**
  - Harder to design / easily go wrong!
  - Extended bass response
  - More efficient that sealed (uses rear radiation)
  - Can have a poor transient response
  - If the velocity of air in the port is too high $\rightarrow$ turbulence $\rightarrow$ chuffing
  - Below box resonance, the velocity can be high – speaker damage

Vented cabinet design: an example

- Design lossless vented cabinet for the SEAS Exotic F8 X1-08 driver

Nominal Impedance	8 Ohms	Voice Coil Resistance	5.7 Ohms
Recommended Frequency Range	30 - 20000 Hz	Voice Coil Inductance	0.07 mH
Short Term Power Handling *	100 W	Force Factor	5.25 N/A
Long Term Power Handling *	35 W	Free Air Resonance	32 Hz
Characteristic Sensitivity (2,83V, 1m)	93.0 dB	Moving Mass	10.0 g
Voice Coil Diameter	26 mm	Air Load Mass In IEC Baffle	1.92 g
Voice Coil Height	7.8 mm	Suspension Compliance	2.5 mm/N
Air Gap Height	12 mm	Suspension Mechanical Resistance	0.57 Ns/m
Linear Coil Travel (p-p)	4.2 mm	Effective Piston Area	222 cm^2
Maximum Coil Travel (p-p)	14 mm	VAS	143 Litres
Magnetic Gap Flux Density	0.8 T	QMS	4.20
Magnet Weight	0.8 kg	QES	0.50
Total Weight	2.6 kg	QTS	0.44

Figure 7: Driver parameters.

Small charts: foolproof vented cabinet design

1. Driver $Q_{TS} = 0.44 \rightarrow \alpha = 1.2$
2. Get acoustic compliance $C_{AS} = C_{MS} S_D^2$

$$C_{AS} = 2.5\times10^{-3}\times(222\times10^{-4})^2 = 1.23\times10^{-6} \quad (23)$$

3. Get box compliance $C_{AB} = C_{AS}/\alpha$

$$C_{AB} = 2.5 \times 10^{-3}/1.2 = 1.027 \times 10^{-6} \quad (24)$$

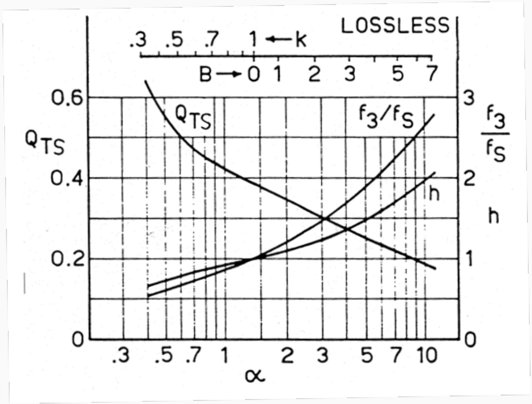


Figure 8: Design chart for **lossless** cabinet.

4. Calculate cabinet volume $V_{AB} = C_{AB}\rho_0 c^2$

$$V_{AB} = 1.027 \times 10^{-6} \times \rho_0 c^2 = 0.146 \quad (25)$$

- In litres $V = V_{AB} \times 1000 = 146$ L

5. Use $\alpha = 1.2$ to get $h = 0.95$ find $f_b = h f_s$

$$f_b = 0.95 \times 32 = 30.4 \quad (26)$$

6. Use f_b and C_{AB} to get vent mass M_{AV}

$$M_{AV} = \frac{1}{\omega_b^2 C_{AB}} = 32.16 \quad (27)$$

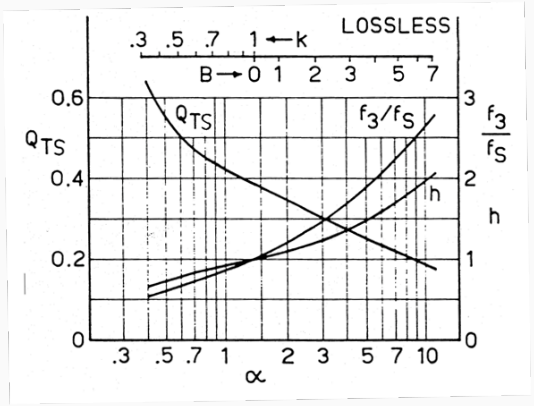


Figure 9: Design chart for **lossless** cabinet.

7. Find vent dimensions that give M_{AV} - vent geometry typically defined by 2 or 3 values - *too many unknowns*
   - Want to keep vent velocity low enough to avoid 'chuffing'
   - Rule of thumb: $S_V \geq 0.8 f_b V_D$ where $V_D = S_D x_{max}$ and x_{max} is the maximum displacement (excursion) of the driver
8. Find vent length from vent mass

$$M_{AV} = \frac{\rho_0 L}{S_V} \rightarrow L = \frac{M_{AV} S_V}{\rho_0} \tag{28}$$

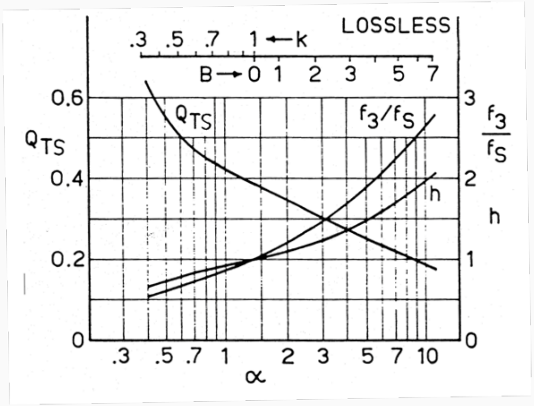


Figure 9: Design chart for **lossless** cabinet.

Vented cabinet: port length end corrections

- **Important:** the volume of air that oscillates extends *beyond* the vent! Extra mass due to radiation loading.

- Find *apparent length* L' and apply end correction
- For circular ports we have simple end correction

$$L = L' - (0.85 + 0.61)\sqrt{S_V/\pi} \tag{29}$$

- Specify vent area using $S_V = 0.8 f_b V_D$

$$S_V = f_b V_D = 30.4 \times (222 \times 10^{-4})\frac{0.014}{2} = 0.00473 \tag{30}$$

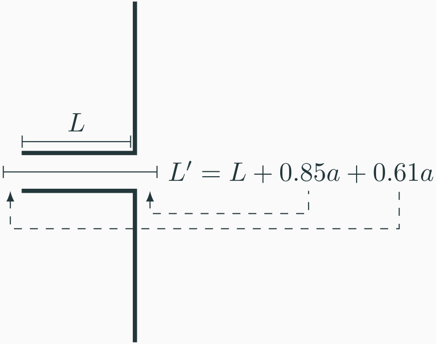


Figure 10: End corrections.

- Solve for the vent length

$$L = L' - (0.85 + 0.61)\sqrt{\frac{0.00473}{\pi}} = 0.0691 \quad (31)$$

8. Determine appropriate dimensions using GR$= 1.618$

$$\text{dim}_1 = \frac{\sqrt[3]{V_B}}{\text{GR}}, \quad \text{dim}_2 = \sqrt[3]{V_B}, \quad \text{dim}_3 = \sqrt[3]{V_B} \times \text{GR} \quad (32)$$

- **Final dimensions**

  Box $0.33 \times 0.53 \times 0.85$ m (width, depth, height)
  Vent: 0.07×0.039 m (length, radius)

- Right cabinet type? EBP $= f_s/Q_{TS} = 73$

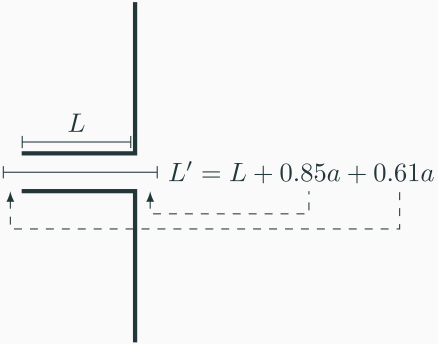


Figure 10: End corrections.

Passive radiator loudspeaker

Passive radiator: another 2 DoF loudspeaker system

- Secondary diaphragm has a mass, suspension compliance and resistance.
- Still have a 2 DoF system... What's changed?
  - We now have an extra compliance due to the secondary diaphragm suspension!

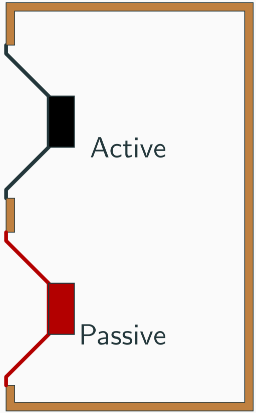


Figure 11: Passive radiator loudspeaker.

Passive radiator: equivalent circuit

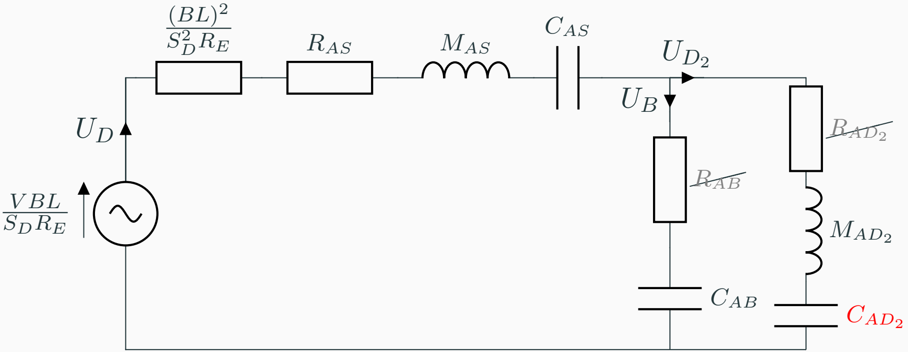


Figure 2: Simplified complete equivalent circuit for passive radiator loudspeaker.

$$F(j\omega) = \frac{1}{\left(1 + \frac{R_{AS}}{j\omega M_{AS}} + \frac{1}{(j\omega)^2 M_{AS} C_{AS}}\right)\left(\frac{1}{(j\omega)^2 M_{AD_2} C_{AB} + \frac{C_{AB}}{C_{AD_2}}} + 1\right) + \frac{1}{(j\omega)^2 M_{AS} C_{AB}}} \quad (33)$$

Next week...

- Performance parameters.
- Electro-mag stuff...

- Reading:
  - Performance parameters: Lecture notes, Sec. 8.4
  - Magnetic motor design: Lecture notes, Chp. 9